Lecture Series: Control and Learning in Games

Chih-Yuan Chiu

Motivation

Modern society increasingly relies upon the shared use of societal infrastructure, such as traffic systems, communication networks, and power grids. Unfortunately, users who access these shared resources are often motivated by selfish objectives and make self-interested decisions at the expense of the overall societal welfare. Indeed, the self-serving behavior of strategic decision-makers has been attributed to traffic gridlock, power outages, communication bottlenecks, and other critical failures in modern cyber-physical systems, phenomena which exemplify the well-established economics concept of the *tragedy of the commons*. Thus, to optimize societal welfare, it is critical to construct accurate and interpretable mathematical models of selfish user behaviors in resource-sharing environments, and design either “soft” incentives or “hard” control commands to influence their behavior.

Course Outline

In this lecture series, we will introduce both classical and more modern techniques in game theory, controls, and optimization to *model* and *control* the strategic behavior of self-interested individuals in multi-agent systems. To provide context, we will discuss emerging challenges in next-generation mobility, such as controller design for autonomous systems deployed in multi-agent interactions, and toll designs for congestion mitigation. However, the introduced methods apply across a broad range of real-world resource-sharing scenarios.

We will begin by reviewing foundational tools in control and optimization theory, both theoretical and algorithmic, that are widely used in game-theoretic modeling and analysis, such as Lyapunov analysis and dynamic programming. Then, we will proceed to study:

1. Classifications of game-theoretic models, e.g., static vs. dynamic games, finite-horizon vs. infinite-horizon games, atomic vs. non-atomic games, population games, etc.;

2. Steady-state strategy profiles in games, e.g., Nash equilibria, correlated equilibria, coarse correlated equilibria, etc.;

3. Zero-sum games, security strategies, upper and lower values, and the Minimax Theorem.

4. Dynamic games, and the role of information patterns in influencing interaction outcomes;

5. Non-atomic and atomic games, and their use in constructing traffic congestion models;

6. Behavioral dynamics in games, and their convergence (or non-convergence) to Nash equilibrium strategies;

7. Mechanism design in games, using traffic congestion management as a running example.

If time permits, we will discuss additional topics, e.g., (1) Dissipativity and passivity models for learning dynamics in population games, or (2) Cooperative game theory and coalition formation.

References

The texts [1–6] below provide the theoretical and algorithmic foundation for much of the material explored in this lecture series. The instructor will also provide lecture notes and/or slides inspired by course material [7–9] and relevant literature readings [10–12] available online.

[1] T. Başar and G. J. Olsder, *Dynamic Noncooperative Game Theory*. SIAM, 1998.

[2] T. Roughgarden, *Twenty Lectures on Algorithmic Game Theory*. Cambridge University Press, 2016. [Online]. Available: `https://timroughgarden.org/f13/f13.html`.

[3] D. Fudenberg and J. Tirole, *Game Theory*. The MIT Press, 1991.

[4] W. H. Sandholm, *Population Games And Evolutionary Dynamics*. The MIT Press, 2010.

[5] S. Sastry, *Nonlinear Systems: Analysis, Stability, and Control*. Springer, 1999.

[6] V. Borkar, *Stochastic Approximation: A Dynamical Systems Viewpoint*. Cambridge University Press, 2008.

[7] J. Marden, *Lecture Notes on Game Theory and Multiagent Systems (ECE 149)*. UCSB, Department of ECE, 2020. [Online]. Available: `https://web.ece.ucsb.edu/~jrmarden/ece149.html`.

[8] A. Ozdaglar, *Lecture Notes on Game Theory with Engineering Applications (6-254)*. MIT, Department of EECS, 2010. [Online]. Available: `https://ocw.mit.edu/courses/6-254-game-theory-with-engineering-applications-spring-2010/`.

[9] M. Manea, *Lecture Notes on Game Theory and Multiagent Systems (14-126)*. MIT, Department of EECS, 2016. [Online]. Available: `https://ocw.mit.edu/courses/14-126-game-theory-spring-2016/`.

[10] L. J. Ratliff, S. A. Burden, and S. S. Sastry, "On the Characterization of Local Nash Equilibria in Continuous Games," *IEEE Transactions on Automatic Control*, vol. 61, no. 8, pp. 2301–2307, 2016. DOI: `10.1109/TAC.2016.2583518`.

[11] E. Mazumdar, L. J. Ratliff, and S. S. Sastry, "On Gradient-Based Learning in Continuous Games," *SIAM Journal on Mathematics of Data Science*, vol. 2, no. 1, pp. 103–131, 2020. DOI: `10.1137/18M1231298`.

[12] C. Maheshwari, K. Kulkarni, M. Wu, and S. S. Sastry, "Dynamic Tolling for Inducing Socially Optimal Traffic Loads," in *2022 American Control Conference (ACC)*, 2022, pp. 4601–4607. DOI: `10.23919/ACC53348.2022.9867193`.